Born Rule, Quantum Gates and Circuits

Introduction to Quantum Computing

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Table of Contents

The Wave Function

Quantum Mechanics Again!

- **Quantum mechanics** is a *mathematical framework* for the development of physical theories.

Some Postulates of QM

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- In general figuring out the Hamiltonian needed to describe a particular physical system is a very difficult problem.

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Can every wave function that resides in Hilbert space good enough to describe the system?

Born's Rule

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Applying Born's rule: $|\psi\rangle = \frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle$

Probability of measuring $|0\rangle = (\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$
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Normalization

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Is this a valid state vector?

$$|a|^2 + |b|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 \tag{1}$$

$$= 1(\text{valid!}) \tag{2}$$

Practise Problems-I

“**Why**,” said the Dodo, “**the best way to explain it is to do it**.” – Lewis Carroll, *Alice’s Adventures in Wonderland*

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Problem 2

Is the vector: $|\psi\rangle = \frac{3}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ capable of describing any quantum system?

a *Yes*

b *No*

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Answer: (b) No

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Problem 3

Check whether the vector: $|\psi\rangle = \frac{3}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ is normalized, or not? If not, normalize the vector.

Hint: Normalized vector = $\frac{1}{\sqrt{a^2+b^2}}|\psi\rangle$

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Quantum Gates

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3. **Z Gate:**
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$$|-\rangle \;-\!\boxed{Z}\!-\; |+\rangle$$

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- Append the qubit(s) to the quantum circuit with measurements or gates applied to it.
  my_circuit.append(cirq.measure(my_qubit))
- Simulate the result of the quantum circuit and get a measurement for our qubit(s).
  sim = cirq.Simulator()
  result = sim.run(my_circuit)
  result

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What will be the final state of a qubit in given circuit:

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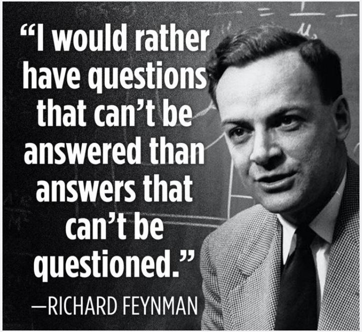
Thank You!


"I would rather have questions that can't be answered than answers that can't be questioned."
—RICHARD FEYNMAN